AC CURRENT GENERATION

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AC CURRENT

- DC currents
  - Special conditions
    - Time-reversal asymmetry
    - Spatial asymmetry
- AC currents
  - Only condition: scatterer's properties change periodically in time
  - 3 different sources of AC current
    - Scattering: varying redistribution of incoming electrons to outgoing channels
    - Periodical change of charge localized on scatterer
    - Bias between reservoirs
  - Note: even DC bias can result in AC current

ADIABATIC AC CURRENT

- Adiabatic regime: $\varpi = \hbar\Omega_0/\delta E \to 0$
- Calculate I_α from the Floquet theorem: $I_\alpha(t) = \sum_{l=-\infty}^{\infty} e^{-il\Omega_0 t} I_{\alpha,l}$
- Using the Fourier harmonics:

$$I_{\alpha,l} = \frac{e}{h}\int_0^\infty dE \left\{ \sum_{\beta=1}^{N_r} \sum_{n=-\infty}^{\infty} S^*_{F,\alpha\beta}(E\ , E_n)\, S_{F,\alpha\beta}(E_l, E_n)\, f_\beta(E_n) - \delta_{l0}\, f_\alpha(E) \right\}$$

- We replace $E_n \to E$ and $n \to -n$ in the sums, use first order adiabatic expansion:

$$S_{F,\alpha\beta}(E_n, E) = S_{\alpha\beta,n}(E) + \frac{n\hbar\Omega_0}{2}\frac{\partial S_{\alpha\beta,n}}{\partial E} + \hbar\Omega_0 A_{\alpha\beta,n} + \mathcal{O}\left(\varpi^2\right)$$

ADIABATIC AC CURRENT

- Calculate the product:

$$S^{*}_{F,\alpha\beta}(E_n, E)\, S_{F,\alpha\beta}(E_{l+n}, E) = S^{*}_{\alpha\beta,n} S_{\alpha\beta,l+n} + \hbar\Omega_0 \left\{ \frac{n}{2} \frac{\partial S^{*}_{\alpha\beta,n}}{\partial E} S_{\alpha\beta,l+n} \right.$$

$$\left. + \frac{(n+l)}{2} \frac{\partial S_{\alpha\beta,n+l}}{\partial E} S^{*}_{\alpha\beta,n} + \left(S^{*}_{\alpha\beta,n} A_{\alpha\beta,l+n} + A^{*}_{\alpha\beta,n} S_{\alpha\beta,n+l} \right) \right\} + \mathcal{O}\left(\varpi^2\right)$$

- And sum up over n:

$$\sum_{n=-\infty}^{\infty} S^{*}_{F,\alpha\beta}(E_n, E)\, S_{F,\alpha\beta}(E_{l+n}, E) = \left(\left| S_{\alpha\beta} \right|^2 \right)_l +$$

$$\frac{i\hbar}{2} \left(- \frac{\partial^2 S^{*}_{\alpha\beta}}{\partial t \partial E} S_{\alpha\beta} + \frac{\partial^2 S_{\alpha\beta}}{\partial t \partial E} S^{*}_{\alpha\beta} \right)_l + \hbar\Omega_0 \left(S^{*}_{\alpha\beta} A_{\alpha\beta} + A^{*}_{\alpha\beta} S_{\alpha\beta} \right)_l + \mathcal{O}\left(\varpi^2\right)$$

3 PARTS OF CURRENT - 1

- We arrive in the first order approximation in Ω_0 at the 3 mentioned terms:

$$I_\alpha(t) = I_\alpha^{(V)}(t) + I_\alpha^{(Q)}(t) + I_\alpha^{(gen)}(t)$$

- First term:

$$I_\alpha^{(V)}(t) = \frac{e}{h} \int_0^\infty dE \sum_{\beta=1}^{N_r} \left|S_{\alpha\beta}(t,E)\right|^2 \left\{f_\beta(E) - f_\alpha(E)\right\}$$

- Exists only if there is a difference of chemical potentials or temperatures in reservoirs (“external bias”)

3 PARTS OF CURRENT - 1

- Has to satisfy unitarity condition → Conservation law

$$\sum_{\alpha=1}^{N_r} I_{\alpha}^{(V)}(t) = 0$$

- This is the same as for the DC current
- Conservation law justifies the separation of this term from total current
- Association with the case of non-zero bias between reservoirs (ΔT, Δμ, ΔU,…)

3 PARTS OF CURRENT - 2

- The second term corresponds to the current contribution of the varying charge Q(t) localized on the scatterer

$$I_{\alpha}^{(Q)}(t) = -e\frac{\partial}{\partial t}\int_{0}^{\infty} dE \sum_{\beta=1}^{N_r} f_{\beta}(E)\,\frac{dN_{\alpha\beta}(t,E)}{dE}$$

- Where:

$$\frac{dN_{\alpha\beta}(t.E)}{dE} = \frac{i}{4\pi}\left\{S_{\alpha\beta}(t,E)\,\frac{\partial S_{\alpha\beta}^{*}(t,E)}{\partial E} - \frac{\partial S_{\alpha\beta}(t,E)}{\partial E}\,S_{\alpha\beta}^{*}(t,E)\right\}$$

- Is the frozen partial Density Of States (DOS), expressed in terms of the frozen scattering matrix elements

3 PARTS OF CURRENT - 2

- Sum up to all leads → Charge conservation law:

$$\sum_{\alpha=1}^{N_r} I_\alpha^{(Q)}(t) + \frac{\partial Q(t)}{\partial t} = 0$$

- Where the charge localized on the scatterer is:

$$Q(t) = e\int_0^\infty dE \sum_{\alpha=1}^{N_r}\sum_{\beta=1}^{N_r} f_\beta(E)\,\frac{dN_{\alpha\beta}(E,t)}{dE}$$

- Other current components should also figure in the charge conservation, but sum is zero
- Really a current due to variation of scatterer charge

3 PARTS OF CURRENT - 3

- First 2 terms were inherent to stationary scatterer
- Third term only if dynamical scatterer

$$I_\alpha^{(gen)}(t) = \int_0^\infty dE \sum_{\beta=1}^{N_r} f_\beta(E) \frac{dI_{\alpha\beta}(t,E)}{dE}$$

- Partial Spectral Current Density: a flow generated by the dynamical scatterer from lead β to lead a

$$\frac{dI_{\alpha\beta}}{dE} = \frac{e}{h}\left(2\hbar\Omega_0 \mathrm{Re}\left[S^*_{\alpha\beta} A_{\alpha\beta}\right] + \frac{1}{2} P\left\{S_{\alpha\beta}, S^*_{\alpha\beta}\right\}\right)$$

3 PARTS OF CURRENT - 3

- Conservation law

$$\sum_{\alpha=1}^{N_r} I_{\alpha}^{(gen)}(t) = 0 \quad \Leftarrow \quad \sum_{\alpha=1}^{N_r} \frac{dI_{\alpha\beta}(t,E)}{dE} = 0$$

- No internal source of charge, only redistribution
- Spectral Current Density (sum up for all β leads):

$$\frac{dI_{\alpha}}{dE} = \frac{e}{h} P\left\{\hat{S}, \hat{S}^{\dagger}\right\}_{\alpha\alpha}$$

- Generated current ⟺ Anomalous scattering matrix (violation of scattering direction reversal symmetry)

EXTERNAL AC BIAS INTRO

- Current flowing through the dynamical scatterer due to reservoirs biased with periodic in time potentials

$$V_{\alpha\beta}(t) \; = \; V_{\alpha\beta}(t + \mathcal{T}) \equiv V_{\alpha}(t) - V_{\beta}(t)$$

- Generated current interferes with bias induced current, Interference Current contribution
- Let the reservoir potentials vary with same frequency as the parameters of the scatterer

$$V_{\alpha}(t) \; = \; V_{\alpha} \cos(\Omega_0 t \; + \; \phi_{\alpha}) \; , \quad \alpha = 1, \ldots, N_r$$

EXTERNAL AC BIAS INTRO

- Phase-coherent transport phenomena ➡ Spatially uniform potentials of electron reservoirs, appears as phase in the incident electron wavefunctions
- Suppose constant μ, independent of potentials
- Schrödinger equation with spatially uniform potential can be integrated

$$i\hbar \frac{\partial \Psi_\alpha}{\partial t} = H_{0,\alpha}\Psi_\alpha + eV_\alpha(t)\Psi_\alpha$$

$$\Psi_\alpha = \Psi_{0,\alpha} e^{-i\hbar^{-1}\int_{-\infty}^{t} dt' eV_\alpha(t')} \qquad \Psi_{0E,\alpha} = e^{-i\frac{E}{\hbar}t}\psi_{E,\alpha}(\vec{r})$$

EXTERNAL AC BIAS INTRO

- With the above defined potential, solution with E energy $(eV_\alpha > 0)$:

$$\Psi_{E,\alpha} = e^{-i\frac{E}{\hbar}t}\bar{\psi}_{E,\alpha}\left(\vec{r}\right)\sum_{n=-\infty}^{\infty} e^{-in\phi_\alpha} J_n\left(\frac{eV_\alpha}{\hbar\Omega_0}\right) e^{-in\Omega_0 t}$$

- Where we used:

$$e^{-iX\sin(\Omega_0 t+\phi_\alpha)} = \sum_{n=-\infty}^{\infty} J_n\left(X\right) e^{-in(\Omega_0 t+\phi_\alpha)}$$

$$C = e^{ieV_\alpha/(\hbar\Omega_0)\,\sin(\Omega_0 t'+\phi_\alpha)|_{t'=-\infty}}$$

$$\bar{\psi}_{E,\alpha}\left(\vec{r}\right) = C\psi_{E,\alpha}\left(\vec{r}\right)$$

- This is a Floquet-type wave function, spatial part only depends on E, but not on sub-band number n

EXTERNAL AC BIAS INTRO

- Therefore Floquet wave function is normalized as stationary wave function

$$\int d^3r \left|\Psi_{E,\alpha}\right|^2 = \int d^3r \left|\psi_{E,\alpha}\right|^2$$

- Proof: $\sum_{n=-\infty}^{\infty} J_n(X)\, J_{n+q}(X) = \delta_{q0}$ using $X = eV_\alpha/(\hbar\Omega_0)$, $q = m - n$, $|C|^2 = 1$

$$\left|\Psi_{E,\alpha}\right|^2 = \left|\psi_{E,\alpha}\right|^2 \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} e^{-i(n-m)\phi_\alpha}\, e^{-i(n-m)\Omega_0 t} J_n(X)\, J_m(X)$$

$$= \left|\psi_{E,\alpha}\right|^2 \sum_{q=-\infty}^{\infty} e^{iq\phi_\alpha}\, e^{iq\Omega_0 t} \sum_{n=-\infty}^{\infty} J_n(X)\, J_{n+q}(X) = \left|\psi_{E,\alpha}\right|^2$$

EXTERNAL AC BIAS INTRO

- The Floquet state with energy E in lead a can be occupied by only one electron
- Its measured energy can be any of $E_n = E + n\hbar\Omega_0$ Floquet energies with probability $J_n^2(eV_\alpha/\hbar\Omega_0)$ (square amplitude)
- Mean energy is the energy of the stationary state

$$E\left[\Psi_{E,\alpha}\right] = \sum_{n=-\infty}^{\infty} E_n J_n^2 = E \sum_{n=-\infty}^{\infty} J_n^2 + \hbar\Omega_0 \sum_{n=1}^{\infty} n\left(J_n^2 - J_{-n}^2\right) = E$$

- Corresponding Fermi distribution only depends on Floquet energy

THANK YOU FOR YOUR ATTENTION!