Alternating current generation II.

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Introducing the creation/annihilation operators

Let $\hat{a}'^{\dagger}_{\alpha}$ be the creation, and $\hat{a}'_{\alpha}$ the annihilation operator in the Floquet state in the reservoir α. By definition they are anti-commuting:

$$\left\langle \hat{a}'^{\dagger}_{\alpha}(E)\,\hat{a}'_{\beta}\left(E'\right)\right\rangle = \delta_{\alpha\beta}\,\delta\left(E-E'\right)f_{\alpha}(E)$$

Where the $f_{\alpha}(E)$ is the Fermi-distribution function.

Assuming, that $\frac{\hbar\Omega_0}{E} \ll 1$ and the oscillating potential is small $eV_{\alpha} \ll E$, we can say, that the scattering of any sub-band of the Floquet state is independent of the scattering of other sub-bands.

Introducing the creation/annihilation operators

If the V_α potential is present in the reservoir, but it is absent in the lead, we can describe the electron in the lead with a wavefunction with fixed energy.

This way we can define the $\hat{a}_\alpha^\dagger$ and the $\hat{a}_\alpha$ creating and annihilating operators in the lead in state $\Psi_\alpha^{(in)}$ as the terms of the $\hat{a}_\alpha'^\dagger$ and $\hat{a}_\alpha'$ operators.

$$\hat{a}_\alpha(E) = \sum_{m=-\infty}^{\infty} e^{-im\phi_\alpha} J_m\left(\frac{eV_\alpha}{\hbar\Omega_0}\right) \hat{a}'_\alpha(E - m\hbar\Omega_0)$$

$$\hat{a}_\alpha^\dagger(E) = \sum_{n=-\infty}^{\infty} e^{in\phi_\alpha} J_n\left(\frac{eV_\alpha}{\hbar\Omega_0}\right) \hat{a}'^\dagger_\alpha(E - n\hbar\Omega_0)$$

Introducing the creation/annihilation operators

The anticommutator of the mentioned operators:

$$\left\{\hat{a}_{\alpha}^{\dagger}(E), \hat{a}_{\beta}(E')\right\} = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} e^{i\phi_{\alpha} n} e^{-i\phi_{\beta} m} J_n\left(\frac{eV_{\alpha}}{\hbar\Omega_0}\right) J_m\left(\frac{eV_{\beta}}{\hbar\Omega_0}\right) \times \left\{\hat{a}'^{\dagger}_{\alpha}(E - n\hbar\Omega_0), \hat{a}'_{\beta}(E' - m\hbar\Omega_0)\right\}$$

$l = n - m$

$$= \delta_{\alpha\beta} \sum_{l=-\infty}^{\infty} e^{i\phi_{\alpha} l} \delta\left(E - E' - l\hbar\Omega_0\right) \sum_{n=-\infty}^{\infty} J_n\left(\frac{eV_{\alpha}}{\hbar\Omega_0}\right) J_{n-l}\left(\frac{eV_{\alpha}}{\hbar\Omega_0}\right)$$

$$= \delta_{\alpha\beta} \sum_{l=-\infty}^{\infty} e^{i\phi_{\alpha} l} \delta\left(E - E' - l\hbar\Omega_0\right) \delta_{l0} = \delta_{\alpha\beta} \delta\left(E - E'\right)$$

Distribution function for incoming eletrons in the lead α

We calculate the distribution of electrons in a lead:

$$\tilde{f}_\alpha(E) = \langle \hat{a}^\dagger_\alpha(E)\,\hat{a}_\alpha(E)\rangle = \sum_{n=-\infty}^{\infty} J_n^2\left(\frac{eV_\alpha}{\hbar\Omega_0}\right) f_\alpha(E - n\hbar\Omega_0)$$

And using this distribution we can calulate the electron current from the reservoir to the scatterer:

$$I_\alpha^{(in)} = -\frac{e}{h}\int_0^\infty dE\,\tilde{f}_\alpha(E) = -\frac{e}{h}\int_0^\infty dE \sum_{n=-\infty}^{\infty} J_n^2\left(\frac{eV_\alpha}{\hbar\Omega_0}\right) f_\alpha(E - n\hbar\Omega_0)$$

$$= -\frac{e}{h}\int_0^\infty dE\,f_\alpha(E)\sum_{n=-\infty}^{\infty} J_n^2\left(\frac{eV_\alpha}{\hbar\Omega_0}\right) = -\frac{e}{h}\int_0^\infty dE\,f_\alpha(E)$$

Introducing the creation/annihilation operators

Let $\hat{b}_\alpha^\dagger$ be the creation, and $\hat{b}_\alpha$ the annihilation operator of the electrons scattered in the lead α. We can express these operators in the terms of $\hat{a}'^{\dagger}_\alpha$ and $\hat{a}'_\alpha$.

$$\hat{b}_\alpha(E) = \sum_{\delta=1}^{N_r} \sum_{n'=-\infty}^{\infty} \sum_{p'=-\infty}^{\infty} S_{F,\alpha\delta}(E, E_{n'})\, e^{-i(n'+p')\phi_\delta} J_{n'+p'}\left(\frac{eV_\delta}{\hbar\Omega_0}\right) \hat{a}'_\delta(E_{-p'})$$

$$\hat{b}^\dagger_\alpha(E) = \sum_{\gamma=1}^{N_r} \sum_{n=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} S^*_{F,\alpha\gamma}(E, E_n)\, e^{i(n+p)\phi_\gamma} J_{n+p}\left(\frac{eV_\gamma}{\hbar\Omega_0}\right) \hat{a}'^{\dagger}_\gamma(E_{-p})$$

Introducing the creation/annihilation operators

These operators are also fermionic operators, and are anticommuting:

$$\left\{\hat{b}_{\alpha}^{\dagger}(E),\hat{b}_{\beta}\left(E'\right)\right\} = \sum_{\gamma=1}^{N_r}\sum_{\delta=1}^{N_r}\sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty}\sum_{n'=-\infty}^{\infty}\sum_{p'=-\infty}^{\infty} e^{i(n+p)\phi_{\gamma}} e^{-i(n'+p')\phi_{\delta}}$$

$$\times J_{n+p}\left(\frac{eV_{\gamma}}{\hbar\Omega_0}\right) J_{n'+p'}\left(\frac{eV_{\delta}}{\hbar\Omega_0}\right) S^{*}_{F,\alpha\gamma}(E,E_n) S_{F,\beta\delta}\left(E',E'_{n'}\right)$$

$$\times\left\{\hat{a}'^{\dagger}_{\gamma}(E-p\hbar\Omega_0),\hat{a}'_{\delta}\left(E'-p'\hbar\Omega_0\right)\right\}$$

Using the equation: $\left\{\hat{a}'^{\dagger}_{\gamma}(E-p\hbar\Omega_0),\hat{a}'^{\dagger}_{\delta}\left(E'-p'\hbar\Omega_0\right)\right\} = \delta_{\gamma\delta}\,\delta\left(E-E'+\left(p'-p\right)\hbar\Omega_0\right)$

Introducing the creation/annihilation operators

$$\left\{\hat{b}_\alpha^\dagger(E), \hat{b}_\beta(E')\right\} = \sum_{\gamma=1}^{N_r} \sum_{m=-\infty}^{\infty} \delta\left(E - E' + m\hbar\Omega_0\right) \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} S_{F,\alpha\gamma}^{*}(E, E_n)$$

$$\times e^{ik\phi_\gamma} S_{F,\beta\gamma}(E_m, E_{n-k}) \sum_{q=-\infty}^{\infty} J_q\left(\frac{eV_\gamma}{\hbar\Omega_0}\right) J_{q+k}\left(\frac{eV_\gamma}{\hbar\Omega_0}\right)$$

Using the properties of the Bessel functions and the unitarity of the Flocquet matrix

$$\left\{\hat{b}_\alpha^\dagger(E), \hat{b}_\beta(E')\right\} = \delta\left(E - E'\right)\delta_{\alpha\beta}$$

Distribution function for scattered eletrons in the lead α

We can calculate the distribution of scattered electrons:

$$f_\alpha^{(out)}(E) = \sum_{\gamma=1}^{N_r} \sum_{n=-\infty}^{\infty} \sum_{n'=-\infty}^{\infty} S_{\alpha\gamma}^{*}(E, E_n)\, S_{\alpha\gamma}(E, E_{n'})\, e^{i(n-n')\phi_\gamma}$$

$$\times \sum_{p=-\infty}^{\infty} J_{n+p}\left(\frac{eV_\gamma}{\hbar\Omega_0}\right) J_{n'+p}\left(\frac{eV_\gamma}{\hbar\Omega_0}\right) f_\gamma(E - p\hbar\Omega_0)$$

Taking the conjugate of this function, and replacing $n \rightarrow n'$. We can see, that this function is real.

Calculating the AC current

Using the already known equation fot the current:

$$\hat{I}_{\alpha}(\omega) = e \int_{0}^{\infty} dE \,\left\{\hat{b}_{\alpha}^{\dagger}(E)\,\hat{b}_{\alpha}(E+\hbar\omega) - \hat{a}_{\alpha}^{\dagger}(E)\,\hat{a}_{\alpha}(E+\hbar\omega)\right\}$$

And substituting back the creation and annihilation operators, we get:

$$I_{\alpha,l} = \frac{e}{h}\int_{0}^{\infty} dE \left\{ \sum_{\gamma=1}^{N_r} \sum_{n=-\infty}^{\infty} \sum_{n'=-\infty}^{\infty} e^{i(n-n'-l)\phi_\gamma} S_{\alpha\gamma}^{*}(E, E_n) S_{\alpha\gamma}(E_l, E_{n'+l}) \right.$$

$$\left. \times \sum_{p=-\infty}^{\infty} J_{n+p}\left(\frac{eV_\gamma}{\hbar\Omega_0}\right) J_{n'+l+p}\left(\frac{eV_\gamma}{\hbar\Omega_0}\right) f_\gamma(E - p\hbar\Omega_0) - \delta_{l0} f_\alpha(E) \right\}$$

Calculating the AC current

We can transform the last equation in a form, where we can see the diference of Fermi-functions, using the equation for Bessel functions:

$$\sum_{n=-\infty}^{\infty} J_n(X)\, J_{n+q}(X) = \delta_{q0}$$

And we get:

$$I_{\alpha,l} = \frac{e}{h}\int_0^{\infty} dE \sum_{\gamma=1}^{N_r}\sum_{p=-\infty}^{\infty}\left\{f_\gamma(E - p\hbar\Omega_0) - f_\alpha(E)\right\}\sum_{n=-\infty}^{\infty}\sum_{n'=-\infty}^{\infty} e^{i(n-n'-l)\phi_\gamma}$$

$$\times S^*_{\alpha\gamma}(E, E_n)\, S_{\alpha\gamma}(E_l, E_{n'+l})\, J_{n+p}\left(\frac{eV_\gamma}{\hbar\Omega_0}\right) J_{n'+l+p}\left(\frac{eV_\gamma}{\hbar\Omega_0}\right)$$

DC current

We can study the DC current as a specific type of the AC, where $l = 0$.

First we express the Floquet scattering matrix in terms of the scattering matrix:

$$S_{\alpha\gamma}(E, E_{n'}) = S_{out,\alpha\gamma,-n'}(E), \quad S^*_{\alpha\gamma}(E, E_n) = S^*_{out,\alpha\gamma,-n}(E)$$

After this we can get:

$$I_{\alpha,0} = \frac{e}{h}\int_0^\infty dE \sum_{\gamma=1}^{N_r} \sum_{p=-\infty}^{\infty} \left\{ f_\beta(E - p\hbar\Omega_0) - f_\alpha(E) \right\}$$

$$\times \left| \left(e^{-i\hbar^{-1}\int_{-\infty}^{t} dt' eV_\gamma(t')} S_{out,\alpha\gamma}(E,t) \right)_p \right|^2$$

DC current

Using the following assumptions,where δE is a characteristic energy scale over which the stationary scattering matrix changes significantly.

$$|eV_\beta| \ll \hbar\Omega_0 \ll \delta E , \quad \forall\beta \quad \text{and} \quad \varpi = \frac{\hbar\Omega_0}{\delta E} \ll 1$$

We can expand the difference of Fermi functions:

$$f_0(E - p\hbar\Omega_0) - f_0(E) \approx \left(-\frac{\partial f_0}{\partial E}\right) p\hbar\Omega_0 + \frac{p^2(\hbar\Omega_0)^2}{2}\frac{\partial^2 f_0}{\partial E^2}$$

DC current

We can expand the current in three parts:

$$I_{\alpha,0} = I_{\alpha,0}^{(pump)} + I_{\alpha,0}^{(rect)} + I_{\alpha,0}^{(int)}$$

Where:

- -$I_{\alpha,0}^{(pump)}$ is generated by the dynamical scatterer in the absence of an oscillating bias;
- -$I_{\alpha,0}^{(rect)}$ *is* due to rectifying of ac currents, produced by the time−dependent potentials, onto the time−dependent conductance.
- -$I_{\alpha,0}^{(int)}$ is due to a mutual influence (an interference) between the currents generated by the scatterer and the currents due to an ac bias.

Pumping and rectifying current

The pumping current was already defined as:

$$I_{\alpha,0} = -i\frac{e}{2\pi}\int_0^\infty dE\left(-\frac{\partial f_0(E)}{\partial E}\right)\int_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\left(\hat{S}(E,t)\frac{\partial\hat{S}^\dagger(E,t)}{\partial t}\right)_{\alpha\alpha}$$

So, now we define the rectifying current as:

$$I_{\alpha,0}^{(rect)} = \frac{e^2}{h}\int_0^\infty dE\left(-\frac{\partial f_0(E)}{\partial E}\right)\int_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\sum_{\gamma=1}^{N_r}V_\gamma(t)\left|S_{\alpha\gamma}(E,t)\right|^2$$

„Interference” current

The „interferece” current can be defined as:

$$I_{\alpha,0}^{(int)} = \frac{e^2}{h}\int_0^\infty dE\left(-\frac{\partial f_0}{\partial E}\right)\int_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\sum_{\gamma=1}^{N_r} V_\gamma(t)$$

$$\times\left(2\hbar\Omega_0 \mathrm{Re}\left[S_{\alpha\gamma}^{*}A_{\alpha\gamma}\right] + \frac{1}{2}P\left\{S_{\alpha\gamma}S_{\alpha\gamma}^{*}\right\}\right)$$

While $I^{(pump)}$ is proportional to Ω_0 and $I^{(rec)}$ is proportional to V_γ, the „interference” current depends on both values.

Properties of diffrent type currents

As we already know, the $I_{\alpha,0}^{(pump)}$ is zero if the the frozen scattering matrix is time-reversal invariant.

The $I_{\alpha,0}^{(rec)}$ is absent if the potentials of all the reservoirs are the same, because we can calculate this current in the following form, where $G_{\alpha\gamma}(t)$ is the element of the frozen conductance matrix.

$$I_{\alpha}^{(rect)} = \int_{0}^{\mathcal{T}} \frac{dt}{\mathcal{T}} \sum_{\gamma=1}^{N_r} G_{\alpha\gamma}(t) \left\{ V_{\gamma}(t) - V_{\alpha}(t) \right\}$$

And even if there is no potential difference or the frozen scattering matrix is time-reversal invariant, the „interference” current still exists.

Thank you for your attention!