Noise of a dynamical scatterer in the adiabatic regime

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Reminder

- The noise of a dynamical scatterer can be expressed as the sum of a thermal noise and shot noise.

$$\mathcal{P}_{\alpha\beta}^{(th)} = \frac{e^2}{h}\int_0^\infty dE\, f_0(E)\,[1 - f_0(E)]\left\{\delta_{\alpha\beta}\left(1 + \sum_{n=-\infty}^{\infty}\sum_{\gamma=1}^{N_r}|S_{F,\alpha\gamma}(E_n, E)|^2\right) - \sum_{n=-\infty}^{\infty}\left(|S_{F,\alpha\beta}(E_n, E)|^2 + |S_{F,\beta\alpha}(E_n, E)|^2\right)\right\},$$

$$\mathcal{P}_{\alpha\beta}^{(sh)} = \frac{e^2}{h}\int_0^\infty dE \sum_{\gamma=1}^{N_r}\sum_{\delta=1}^{N_r}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\sum_{p=-\infty}^{\infty}\frac{[f_0(E_n) - f_0(E_m)]^2}{2}\cdot S_{F,\alpha\gamma}^{*}(E, E_n)S_{F,\alpha\delta}(E, E_m)S_{F,\beta\delta}^{*}(E_p, E_m)S_{F,\beta\gamma}(E_p, E_n).$$

Motivation

- Calculate the thermal noise in the adiabatic regime.
- Examine the temperature dependence of the shot noise.
- Calculate the dependence of the total noise on the pumping frequency and the dominating noise-type at different temperatures.

Thermal noise in the adiabatic regime

- The thermal noise up to linear in pumping frequency terms:

$$\mathcal{P}_{\alpha\beta}^{(th)} = \mathcal{P}_{\alpha\beta}^{(th,0)} + \mathcal{P}_{\alpha\beta}^{(th,\Omega_0)},$$

where

$$\begin{aligned}\mathcal{P}_{\alpha\beta}^{(th,0)} =& k_B T \int_0^\infty dE \left(-\frac{\partial f_0}{\partial E}\right) \int_0^\tau \frac{dt}{\tau} \cdot \\ & \cdot \frac{e^2}{h}\left(2\delta_{\alpha\beta} - |S_{\alpha\beta}(t,E)|^2 - |S_{\beta\alpha}(t,E)|^2\right), \\ \mathcal{P}_{\alpha\beta}^{(th,\Omega_0)} =& k_B T \int_0^\infty dE \left(-\frac{\partial f_0}{\partial E}\right) \int_0^\tau \frac{dt}{\tau} \cdot \\ & \cdot e\left(\delta_{\alpha\beta}\frac{dI_\alpha(t,E)}{dE} - \frac{dI_{\alpha\beta}(t,E)}{dE} - \frac{dI_{\beta\alpha}(t,E)}{dE}\right).\end{aligned}$$

Quasi-equilibrium noise

- Using the time-averaged frozen conductance matrix:

$$\bar{G} = \int_0^{\tau} \frac{dt}{\tau} \hat{G}(t),$$

the zeroth order term can be written as

$$\mathcal{P}^{(th,0)}_{\alpha\beta} = k_B T \left(2\delta_{\alpha\beta} G_0 - \bar{G}_{\alpha\beta} - \bar{G}_{\beta\alpha} \right) .$$

- This has the same form as the equilibrium noise in the stationary case, so it's called the quasi-equilibrium noise.

Non-equilibrium thermal noise

- The first order term is linear in temperature, so it's a thermal noise.
- It depends on the current generated by the dynamical scatterer, so it's non-equilibrium.
- Since the spectral current powers are linear in pumping-frequency,

$$\mathcal{P}_{\alpha\beta}^{(th,\Omega_0)} \sim \Omega_0.$$

Low-temperature shot noise

- The Floquet scattering matrix to the zeroth order in Ω_0 :

$$\hat{S}_F(E_m, E_p) = \hat{S}_{m-p}(E) + \mathcal{O}(\Omega_0).$$

- In the adiabatic regime the frozen scattering matrix is constant over the scale of order $\hbar\Omega_0$.
- At low temperatures, $k_B T \ll \hbar\Omega_0$, the remaining integral is:

$$\int_0^{\infty} dE\,(f_0(E_n) - f_0(E_m))^2 = \hbar\Omega_0\,|m-n|\,.$$

- This way the low-temperature shot noise:

$$\mathcal{P}_{\alpha\beta}^{(sh)} = \frac{e^2\Omega_0}{4\pi} \sum_{\gamma,\delta=1}^{N_r} \sum_{\substack{n,m=-\infty \\ p=-\infty}}^{\infty} |m-n|\, S^*_{\alpha\gamma,-n}(\mu) S_{\alpha\delta,-m}(\mu) S^*_{\beta\delta,p-m}(\mu) S_{\beta\gamma,p-n}(\mu).$$

Low-temperature shot noise

- For any $X_{n,m}$ quantity, where $q = m - n$:

$$\sum_{m=-\infty}^{\infty} |m-n|\, X_{m,n} = \sum_{m=-\infty}^{n-1} (n-m)\, X_{m,n} + \\ + \sum_{m=n+1}^{\infty} (m-n)\, X_{m,n} = \sum_{q=1}^{\infty} q\left(X_{-q+n,n} + X_{q+n,n}\right),$$

- For the Fourier coefficients of two periodic functions:

$$\sum_{n=-\infty}^{\infty} A_n (B_{n+q})^* = \{AB^*\}_{-q}, \quad \sum_{n=-\infty}^{\infty} A_{n+q} (B_n)^* = \{AB^*\}_{q},$$

Low-temperature shot noise

- Using these the low-temperature shot noise can be rewritten as:

$$\mathcal{P}_{\alpha\beta}^{(sh)} = \frac{e^2\Omega_0}{4\pi} \sum_{\gamma,\delta=1}^{N_r} \sum_{q=1}^{\infty} q \left[\left\{ S_{\alpha\gamma}^{*}(\mu) S_{\alpha\delta}(\mu) \right\}_{-q} \left\{ S_{\beta\gamma}^{*}(\mu) S_{\beta\delta}(\mu) \right\}_{q} + \right.$$
$$\left. + \left\{ S_{\alpha\gamma}^{*}(\mu) S_{\alpha\delta}(\mu) \right\}_{q} \left\{ S_{\beta\gamma}^{*}(\mu) S_{\beta\delta}(\mu) \right\}_{-q} \right].$$

- It can be easily shown, that this satisfies the symmetry

$$\mathcal{P}_{\alpha\beta}^{(sh)} = \mathcal{P}_{\beta\alpha}^{(sh)}.$$

High-temperature shot noise

- At high temperature, $k_B T \gg \hbar\Omega_0$, the difference of the Fermi-functions expanded to the first non-zero term:

$$f_0(E_n) - f_0(E_m) = \hbar\Omega_0 \frac{\partial f_0(E)}{\partial E}(n-m).$$

- Using the identities for the Fourier coefficients, the high-temperature shot noise can be written as:

$$\mathcal{P}^{(sh)}_{\alpha\beta} = \frac{e^2}{4\pi}\hbar\Omega_0^2 \int_0^\infty dE \left(\frac{\partial f_0}{\partial E}\right)^2 \sum_{q=-\infty}^{\infty} q^2 \cdot$$

$$\cdot \sum_{\gamma,\delta=1}^{N_r} \left\{S^*_{\alpha\gamma}(E)S_{\alpha\delta}(E)\right\}_q \left\{S^*_{\beta\gamma}(E)S_{\beta\delta}(E)\right\}_{-q}.$$

- At high temperatures the shot noise is quadratic in pumping-frequency.

Shot noise in wide temperature range

- Let δE be the interval over which the scattering matrix changes significantly.
- If $\hbar\Omega_0, k_B T \ll \delta E$, the scattering matrix can be approximated with it's value at $E = \mu$.
- Using this, the remaining integral can be calculated analytically:

$$\int_0^\infty dE \left[f_0(E_n) - f_0(E_m)\right]^2 = (m-n)\hbar\Omega_0 \coth\left(\frac{(m-n)\hbar\Omega_0}{2k_B T}\right) - 2k_B T.$$

Shot noise in wide temperature range

- The shot noise:

$$\mathcal{P}_{\alpha\beta}^{(sh)} = \frac{e^2}{h} \sum_{q=-\infty}^{\infty} F(q\hbar\Omega_0, k_B T) \cdot \cdot \sum_{\gamma,\delta=1}^{N_r} \left\{ S_{\alpha\gamma}^{*}(E) S_{\alpha\delta}(E) \right\}_q \left\{ S_{\beta\gamma}^{*}(E) S_{\beta\delta}(E) \right\}_{-q},$$

where

$$F(x,y) = \frac{x}{2} \coth\left(\frac{x}{2y}\right) - y = \begin{cases} \frac{|q|\hbar\Omega_0}{2}, \text{if} k_B T \ll \hbar\Omega_0, \\ \frac{(q\hbar\Omega_0)^2}{12 k_B T}, \text{if} k_B T \gg \hbar\Omega_0. \end{cases}$$

- This gives the previous results for the low and high temperature cases.

Pumping-frequency dependece of the total noise

- The part of the total noise dependent on Ω_0 can be written as:

$$\delta\mathcal{P}_{\alpha\beta}^{(\Omega_0)} = \mathcal{P}_{\alpha\beta}^{(sh)} + \mathcal{P}_{\alpha\beta}^{(th,\Omega_0)}.$$

- These can be estimated as:

$$\mathcal{P}_{\alpha\beta}^{(th,\Omega_0)} \sim k_B T \frac{\hbar\Omega_0}{\delta E}, \qquad \mathcal{P}_{\alpha\beta}^{(sh)} \sim \frac{(\hbar\Omega_0)^2}{k_B T}.$$

- Then the ratio is

$$\frac{\mathcal{P}_{\alpha\beta}^{(sh)}}{\mathcal{P}_{\alpha\beta}^{(th,\Omega_0)}} \sim \frac{\hbar\Omega_0 \delta E}{(k_B T)^2}.$$

Pumping-frequency dependece of the total noise

- So with increasing temperature the total noise of a dynamical scatterer:

$$\delta\mathcal{P}_{\alpha\beta}^{(\Omega_0)} \sim \frac{e^2}{2h}\begin{cases} \hbar\Omega_0, & \text{if } k_B T \ll \hbar\Omega_0, \\ \frac{(\hbar\Omega_0)^2}{6k_B T}, & \text{if } \hbar\Omega_0 \ll k_B T \ll \sqrt{\hbar\Omega_0\delta E}, \\ \hbar\Omega_0\frac{k_B T}{\delta E}, & \text{if } \sqrt{\hbar\Omega_0\delta E} \ll k_B T. \end{cases}$$

- The noise is linear in pumping-frequency at low and high temperatures, but for different physical reasons:
  - at low temperature due to the shot noise,
  - at high temperature due to the thermal noise.
- At an intermediate temperature range the noise is quadratic due to the shot noise.

Thank you for your attention!