Direct Current Generation
part I

Msc seminar

Balassa Gábor

Table of contents

DC current

- Steady particle flow in the leads connecting a scatterer to the reservoirs.
- Periodic excitation (without bias) -> Dc
  - Quantum pump effect (linear, quadratic, …)
- Distribution functions characterizes the intensity
- Basic assumption 1: the reservoirs are in equilibrium
  - Fermi distribution

- Direct current = difference of particle flows in the two directions times the electric charge
- Charge conservation must be satisfied
- Basic assumption 2:
  - μ and T are the same at all reservoirs
- But the scattering on the dynamical sample is non-equilibrium

$$I_{\alpha,0} = \frac{e}{h} \int_{0}^{\infty} dE \left\{ f_{\alpha}^{(out)}(E) - f_{\alpha}(E) \right\}.$$

$$f_{\alpha}^{(out)}(E) = \sum_{n=-\infty}^{\infty} \sum_{\beta=1}^{N_r} \left| S_{F,\alpha\beta}(E, E_n) \right|^2 f_{\beta}(E_n).$$

Energy shift due to dynamic (periodic) scatterer

$$E_n \equiv E + n\hbar\Omega_0 \le \mu_0 .$$

$$f_\alpha^{(out)}(E) = \sum_{n=-\infty}^{\left[\frac{\mu-E}{\hbar\Omega_0}\right]} \sum_{\beta=1}^{N_r} \left|S_{F,\alpha\beta}(E, E_n)\right|^2 = \begin{cases} <1, & E<\mu_0, \\ >0, & E>\mu_0, \end{cases}$$

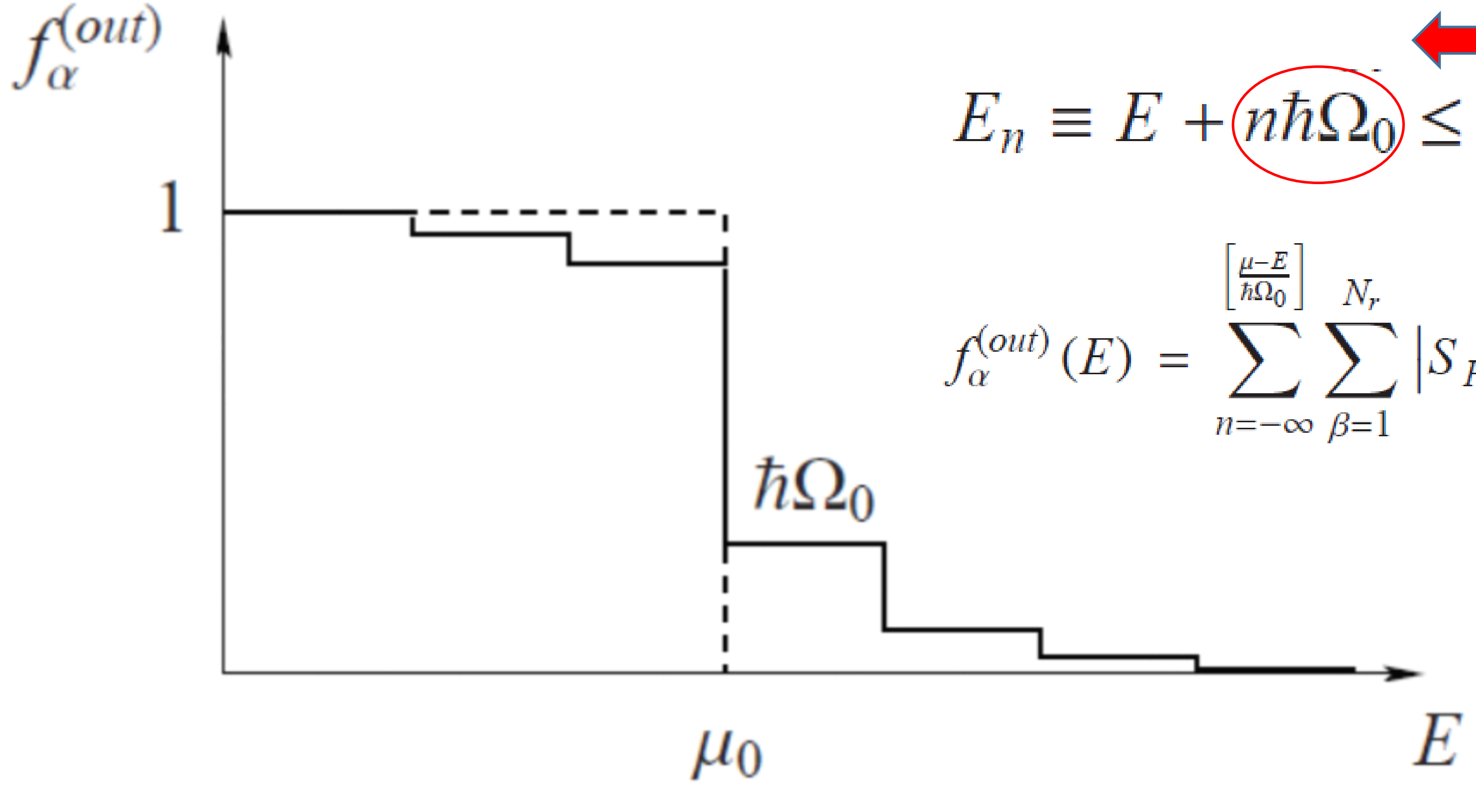

Figure 4.1: The non-equilibrium distribution function, $f_\alpha^{(out)}(E)$, for scattered electrons at zero temperature is shown schematically. The step width is $\hbar\Omega_0$. The zero-temperature Fermi function is shown by dashed line.

Adiabatic regime

- Small pumping frequency
- Expanding the difference of the distribution functions
- Zero-order adiabatic approximation of the scattering matrix

$$I_{\alpha,0} = \frac{e\Omega_0}{2\pi} \int_0^\infty dE \left(-\frac{\partial f_0}{\partial E}\right) \sum_{\beta=1}^{N_r} \sum_{n=1}^{\infty} n \left\{ \left|S_{\alpha\beta,n}(E)\right|^2 - \left|S_{\alpha\beta,-n}(E)\right|^2 \right\}$$

Fourier coeff. of the frozen scattering matrix element

n>0 : emission
n<0 : absorption

- The current is non-zero if: $\hat{S}(t,E) \neq \hat{S}(-t,E)\,.$
  - (time reversal symmetry is broken!)
- Dynamical breaking of the symmetry e.g. (two parameters varying with the same frequency but shifted phase)

$$p_1(t) = p_{1,0} + p_{1,1}\cos(\Omega_0 t)\,,$$

$$p_2(t) = p_{2,0} + p_{2,1}\cos(\Omega_0 t + \varphi)\,.$$

$t \rightarrow -t$ ⟺ $\varphi \rightarrow -\varphi$

- Compact form with iFT:

$$I_{\alpha,0} = -i\frac{e}{2\pi}\int_0^\infty dE\left(-\frac{\partial f_0(E)}{\partial E}\right)\int_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\left(\hat{S}(E,t)\frac{\partial \hat{S}^\dagger(E,t)}{\partial t}\right)_{\alpha\alpha}$$

  - Charge conservation: $$\sum_{\alpha=1}^{N_r} I_{\alpha,0} = 0\,.$$

  - Current at T=0 & finite T
  - T=0 limit :

$$-\partial f_0/\partial E = \delta(E-\mu). \quad\Longrightarrow\quad I_{\alpha,0} = -i\frac{e}{2\pi}\int_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\left(\hat{S}(t,\mu)\frac{\partial \hat{S}^\dagger(t,\mu)}{\partial t}\right)_{\alpha\alpha}.$$

- Special case:
  - Two leads
  - Scattering matrix

Reflection coeff.

$$\hat{S} = e^{i\gamma}\begin{pmatrix} \sqrt{R}\,e^{-i\theta} & i\sqrt{1-R}\,e^{-i\phi} \\ i\sqrt{1-R}\,e^{i\phi} & \sqrt{R}\,e^{i\theta} \end{pmatrix}$$

  - Coefficients are periodic
  - Direct Current:

$$I_0 = \frac{e}{4\pi}\int^{\infty} dE\left(-\frac{\partial f_0(E)}{\partial E}\right)\int_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\left\{R(t)\frac{\partial\theta(t)}{\partial t} + T(t)\frac{\partial\phi(t)}{\partial t}\right\}$$

Phase dependence->QM behaviour

P1, P2 – scattering matrix parameters

$\mathfrak{L}$. - closed trajectory

$$d\hat{S}^{\dagger} = \frac{\partial\hat{S}^{\dagger}}{\partial p_1}dp_1 + \frac{\partial\hat{S}^{\dagger}}{\partial p_2}dp_2\,. \quad \Longrightarrow \quad I_{\alpha,0} = -i\,\frac{e\Omega_0}{4\pi^2}\oint_{\mathfrak{L}}\left(\hat{S}\,d\hat{S}^{\dagger}\right)_{\alpha\alpha}.$$

• Green's theorem ->

$$I_{\alpha,0} = \mathfrak{F} \frac{e\Omega_0}{2\pi^2} \operatorname{Im} \left(\frac{\partial \hat{S}}{\partial p_1} \frac{\partial \hat{S}^\dagger}{\partial p_2} \bigg|_{p_i = p_{i,0}} \right)_{\alpha\alpha},$$

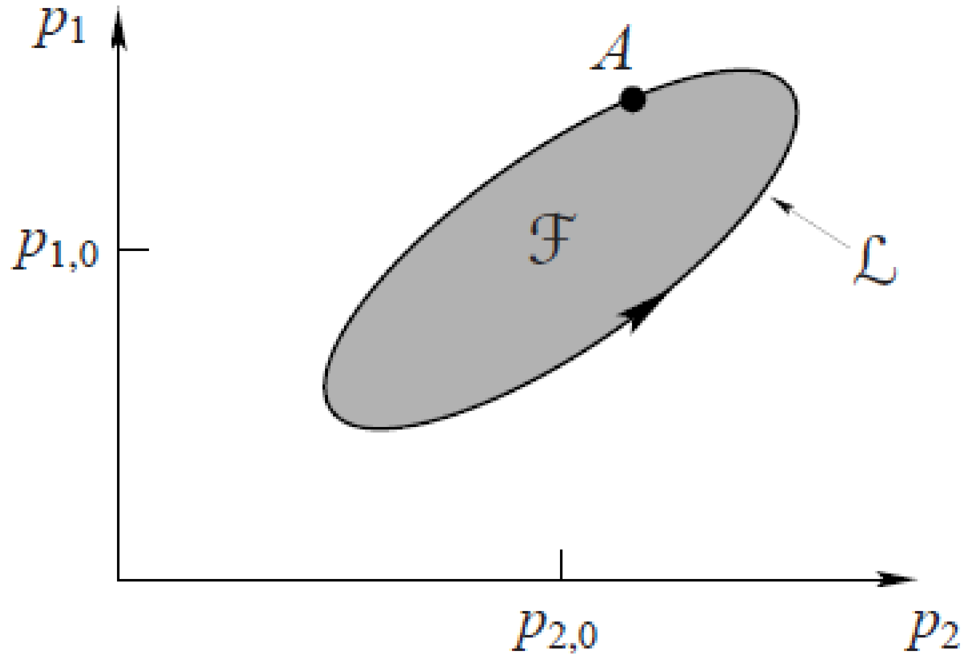

Figure 4.2: During one period the point $A(t)$ with coordinates $(p_1(t), p_2(t))$ follows a trajectory $\mathfrak{L}$. $\mathfrak{F}$ stands for a surface area. The arrow indicates a movement direction for $\varphi > 0$.

- If the parameters vary with small amplitudes then we can keep the derivatives of the scattering matrix elements constant → calculate the derivatives at $p_i = p_{i,0}$.

- If the area is nonzero then the dc current in the adiabatic regime is non-zero.

- Under time reversal the direction of motion of point A changes by its opposite →F->-F

- For some p1,p2 parameters the derivatives can vanish → the dc current vanishes. Frozen scattering matrix→doesn't connect directly to the driving→the stationary characteristic of the scatterer is important!

- Spatially asymetric scatterer is needed (necessary condition).

- Quadratic dependence: $\mathcal{F} = \pi p_{1,1} p_{2,1} \sin(\varphi)$
- The pump effect is nonlinear.
- Phase change→time reversal→current direction is changed.
- Adiabatic regime $\hbar\Omega_0 \ll \delta E$, → the generated current can be represented as the sum of contributions due to electrons with different energies → spectral density of the generated currents $dI_\alpha(t,E)/dE$.

$$I_{\alpha,0} = \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \int_0^{\infty} dE\, f_0(E)\, \frac{dI_\alpha(t,E)}{dE}$$

- Spectral density function: Diagonal elements

$$\frac{dI_\alpha(t,E)}{dE} = \frac{e}{h}P\{\hat{S},\hat{S}^\dagger\}_{\alpha\alpha} \equiv i\frac{e}{2\pi}\left(\frac{\partial\hat{S}}{\partial t}\frac{\partial\hat{S}^\dagger}{\partial E} - \frac{\partial\hat{S}}{\partial E}\frac{\partial\hat{S}^\dagger}{\partial t}\right)_{\alpha\alpha}$$

- Conservation law: $$\sum_{\alpha=1}^{N_r}\frac{dI_\alpha}{dE} = \frac{e}{h}\sum_{\alpha=1}^{N_r}P\{\hat{S},\hat{S}^\dagger\}_{\alpha\alpha} = 0\,.$$

- Problem:
  - No DC if the phase difference is zero.
  - No DC if only one parameter varies in time.
- But the dynamical scatterer can generate quadratic, … dc as well: $I_{\alpha,0} \sim \Omega_0^2 \leftarrow$ non adiabatic

- Expand further → $$I_{\alpha,0} = \frac{e}{2\pi}\int_0^\infty dE\left(-\frac{\partial f_0}{\partial E}\right)\int_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\,\mathrm{Im}\left\{\hat{S}\,\frac{\partial \hat{S}^\dagger}{\partial t} + 2\hbar\Omega_0\hat{A}\,\frac{\partial \hat{S}^\dagger}{\partial t}\right\}_{\alpha\alpha}$$

Linear

- If S(t)=S(-t) → linear term vanishes.

$$I_{\alpha,0}^{(2)} = \frac{e\hbar\Omega_0}{\pi}\int_0^\infty dE\left(-\frac{\partial f_0}{\partial E}\right)\int_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\,\mathrm{Im}\left\{\hat{A}\,\frac{\partial \hat{S}^\dagger}{\partial t}\right\}_{\alpha\alpha}.$$

- Conservation law: $$\int_0^{\mathcal{T}}\frac{dt}{\mathcal{T}}\,\mathrm{Im}\,\mathrm{Tr}\left(\hat{A}(t,E)\,\frac{\partial \hat{S}^\dagger(t,E)}{\partial t}\right) = 0\,.$$

Summary

- DC formation without any bias.
- Equilibrium before scattering.
- Dynamical and stationary conditions to the scatterer:
  - Broken time reversal symmetry
  - Spatial asymmetry
- Linear and quadratic pumping.