Quantum pump effect

Nagy Dániel Bálint

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Reminder

- Current generated by a dinamical scatterer can have a dc component without bias between the reservoirs.
- The necessary conditions are the time-reversal and spatial-inversion asymmetry of the scatterer.
- Dc current can exist in the adiabatic regime if multiple parameters of the frozen scaterring matrix are varied.

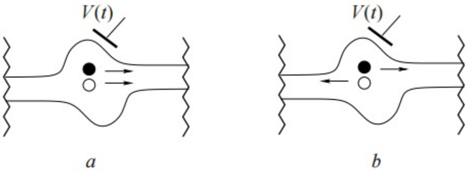
Motivation

- Give a simple argument for the existence of dc current without bias.
- Outline a physical mechanism that can produce the necessary asymmetries.
- Show a possibility for dc current in the adiabatic regime by varying only a single parameter.

Quasi-particle picture

- Quasi-particles are used instead of real particles.
- Particles with $E_n > \mu_0$ are quasi-electrons.
- Empty states with $E_n < \mu_0$ are holes.
- The reservoirs are assumed to be at $T_\alpha = 0K$ with the same chemical potentials.
- Then no quasi-particles flow incident to the scatterer. The dynamical scatterer is the only source of quasi-particles.

Source of quasi-particles



- A real electron can absorb $\Delta E = n\hbar\Omega_0$ from the dynamical scatterer.
- A quasi-electron and a hole is created (total charge remains the same).
- They can leave two ways:
  - a through the same lead, no current generated,
  - b through different leads, a current pulse is generated in two leads.

Role of asymmetry

- If the scatterer has time and spatial symmetry, the probability of quasi-electrons and holes scaterred to a lead is the same.
- Averaged over a long time, the current is zero in all leads, there is no dc current.
- With asymmetry there is a possibility for a dc current, even without bias.

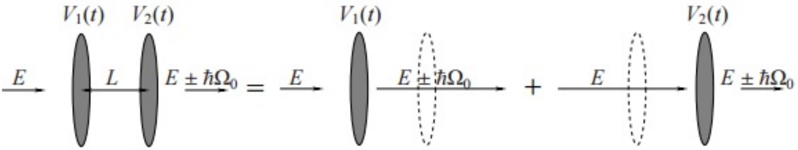
The physical mechanism

- The cause of asymmetry is an interference of photon-assisted scattering amplitudes.
- A simple example is a one-dimensional scatterer with two potentials with a distance of L:

$$V_1(t) = 2V\cos(\Omega_0 t + \phi_1), \qquad V_2(t) = 2V\cos(\Omega_0 t + \phi_2).$$

- The amplitudes are assumed to be small, so only single-photon processes are relevant.

Propagation through the scatterer



- When an electron with E energy falls on the scatterer, it can:
  - a not interact with the potentials. $E^{(out)} = E$
  - b absorb one energy quantum $E^{(out)} = E + \hbar\Omega_0$
  - c emit one energy quantum $E^{(out)} = E - \hbar\Omega_0$
- The total transmission probability:

$$T = T_0(E, E) + T^{(+)}(E + \hbar\Omega_0, E) + T^{(-)}(E - \hbar\Omega_0, E)$$

Calculating the transmissions

- T_0 isn't interesting, since it doesn't depend on the direction.
- For $T^{(+)}$ there are two possibilities, absorbing an energy quantum from $V_1(t)$ or $V_2(t)$.
- The final state is the same, so the amplitudes are added together:

$$T^{(+)} = \left| \mathcal{A}^{(1,+)} + \mathcal{A}^{(2,+)} \right|^2$$

- The $\mathcal{A}^{(j,+)}$ amplitudes are the products of $\mathcal{A}^{(free)} = e^{ikL}$ and $\mathcal{A}_j^{(+)} = \kappa V e^{-i\phi_j}$, where κ is a constant.
- The transmission will depend on the direction of the propagation: $T_{\leftarrow}^{(+)} \neq T_{\rightarrow}^{(+)}$.

Calculating the transmissions

- If $\hbar\Omega_0 \ll E$, $k(E_+) \approx kL + \Omega_0\tau$, where $\tau = Lm/(\hbar k)$
- For an electron propagating right:

$$\mathcal{A}_{\rightarrow}^{(1,+)} = \kappa V e^{-i\phi_1} e^{i(kL+\Omega_0\tau)}, \qquad \mathcal{A}_{\rightarrow}^{(2,+)} = e^{ikL}\kappa V e^{-i\phi_2},$$

$$T_{\rightarrow}^{(+)} = 2\kappa^2 V^2 \left\{1 + \cos\left(\phi_1 - \phi_2 - \Omega_0\tau\right)\right\}.$$

- For an electron propagating left:

$$\mathcal{A}_{\leftarrow}^{(1,+)} = e^{ikL}\kappa V e^{-i\phi_1}, \qquad \mathcal{A}_{\leftarrow}^{(2,+)} = \kappa V e^{-i\phi_2} e^{i(kL+\Omega_0\tau)}$$

$$T_{\leftarrow}^{(+)} = 2\kappa^2 V^2 \left\{1 + \cos\left(\phi_1 - \phi_2 + \Omega_0\tau\right)\right\}.$$

Calculating the dc current

- The two transmissions are indeed different as mentioned before, the difference is:

$$\Delta T^{(+)} = 4\kappa^2 V^2 \sin\left(\Delta\phi\right) \sin\left(\Omega_0 \tau\right).$$

- With a similar calculation, it can be shown for the emission, that $\Delta T^{(-)} = \Delta T^{(+)}$.
- If the electron flow with intensity I_0 falls upon the scatterer from both sides:

$$I_{dc} = I_0 \left(\Delta T^{(+)} + \Delta T^{(-)}\right) = 2 I_0 \Delta T^{(+)}$$

The asymmetries in the current

$$I_{dc} = 8 I_0 \kappa^2 V^2 \sin(\Delta\phi) \sin(\Omega_0 \tau)$$

- The current depends on two factors:
  - $\Delta\phi$ difference between the phases of the potentials. This represents the breaking of the time-reversal invariance.
  - $\Omega_0 \tau = \Omega_0 L / v$ dynamical phase. This represents the spatial asymmetry due to two different potentials at distance L.
- The equation for the dc current shows that both symmetries must be broken for it to exist.

Single-parameter adiabatic current

- Varying multiple parameters of the scattering matrix can result in dc current in the adiabatic regime.
- Variation of a single parameter can result in an at least quadratic in pumping frequency dc current.
- Using a parameter with special topology there can be adiabatic dc current even with only a single parameter.

Periodic scattering matrix

- $S(t) = S(p(t))$ scattering matrix has only one time-dependent parameter.
- If $S(t)$ is periodic in time, there are two cases:
  - i $p(t)$ is also periodic in time.
  - ii p is an angle, i.e. $S(p)$ is periodic itself. In this case $p \sim t$.
- It can be shown, that in this case the time-dependent current has the form:

$$I_\alpha(t) = eC_{\alpha\alpha}(p)\frac{\partial p}{\partial t},$$

where

$$C_{\alpha\alpha}(p) = -\frac{i}{2\pi}\left(\hat{S}\frac{\partial \hat{S}}{\partial p}\right)_{\alpha\alpha}.$$

Dependence on the parameter

- For small p, $C_{\alpha\alpha}(p) \approx C_{\alpha\alpha}(0)$.
  - i Since $p(t)$ is periodic, the current will also be periodic without a dc component.
  - ii The derivative of $p(t)$ is a constant, so there can be a dc component if $C_{\alpha\alpha}(0) \neq 0$.
- For large p:
  - i $C_{\alpha\alpha}(p)$ can be written as a Taylor series of p. Each term in the current will be periodic in time.
  - ii $C_{\alpha\alpha}(p)$ can be written as a Fourier series of p. If the constant term is non-zero there will be a dc current.
- So if the $p = \Omega_0 t$ parameter is a cyclical coordinate, then there can be an $I_\alpha \sim \Omega_0$ dc current with adiabatic pumping.

Thank you for your attention!